Partner switching stabilizes cooperation in coevolutionary prisoner's dilemma

Feng Fu,[1,2,*] Te Wu,[2,†] and Long Wang[2,‡]

[1]*Program for Evolutionary Dynamics, Harvard University, Cambridge, Massachusetts 02138, USA*

[2]*Center for Systems and Control, State Key Laboratory for Turbulence and Complex Systems, College of Engineering, Peking University, Beijing 100871, China*

Previous studies suggest that cooperation prevails when individuals can switch their interaction partners quickly. However, it is still unclear how quickly individuals should switch adverse partners to maximize cooperation. To address this issue, we propose a simple model of coevolutionary prisoner's dilemma in which individuals are allowed to either adjust their strategies or switch their defective partners. Interestingly, we find that, depending on the game parameter, there is an optimal tendency of switching adverse partnerships that maximizes the fraction of cooperators in the population. We confirm that the stabilization of cooperation by partner switching remains effective under some situations, where either normalized or accumulated payoff is used in strategy updating, and where either only cooperators or all individuals are privileged to sever disadvantageous partners. We also provide an extended pair approximation to study the coevolutionary dynamics. Our results may be helpful in understanding the role of partner switching in the stabilization of cooperation in the real world.

I. INTRODUCTION

The prisoner's dilemma [1] has been widely used to study the evolution of cooperation [2] among self-regarding individuals based on evolutionary game theory [3,4]. In this simple game, two players have a choice between cooperation and defection. If both players cooperate they receive R, which is greater than what they would receive if they both defected (P). The highest payoff T is attained by the individual that defects against a cooperator, while that cooperator receives the lowest payoff S (i.e., $T > R > P > S$). Here, we shall adopt a donor-recipient version of this game: a cooperator C pays a cost c and provides a benefit b for another individual ($b > c$). A defector D has no cost and does not provide benefits. Thus, $R = b - c$, $S = -c$, $T = b$, and $P = 0$. The prisoner's dilemma depicts the conflict of interest between what is best for the individual (defection) and what is best for the group (cooperation), and thus creates a social dilemma [5]. In order for the population to escape this dilemma, specific mechanisms for the evolution of cooperation are needed (see Ref. [2] for a recent review).

Inspired by spatial games [6,7], evolutionary games on graphs [8] have increasingly been attracting interest from different fields [9–20] as significant extensions of traditional evolutionary game theory focusing on well-mixed populations [21–24] (see Ref. [25], and references therein). In games on graphs, individuals are situated on the vertices of a graph. The edges indicate interactions (and competition) among individuals. The evolutionary success of an individual is determined by its payoff accrued in pairwise interactions with its neighbors. In this context, network topology naturally plays a decisive role in the evolution of cooperation

[16,19]. Furthermore, it has been found that cooperation significantly increases with social diversity, i.e., heterogeneity in number of interaction partners [19,20], except in cases where there is a cost to participate in the game [26] or the payoff is normalized by the number of neighbors [27–29].

Previous work focuses considerable attention on games on static (fixed) networks [25]. In most real-world situations, however, the topology of the network changes in response to the state of the network and the state changes in response to the topology. In this way a feedback loop between the state and the topology of the network is formed. This feedback loop leads to an intriguing interplay between network adaptation and the nodes' dynamics. Very recently, there has also been a growing interest in the dynamics of adaptive (or coevolutionary) networks (see Ref. [30] for a latest review). In addition, a number of papers studying games on dynamical graphs have recently appeared [31–40]. The main finding from these papers is that the entangled evolution of individual strategies and network structure can give rise to robust cooperation in social networks [31,34,35]. This is supported by empirical evidence that shows partner switching causes cooperative behavior in cleaning fishes [41] and in human groups [42].

Motivated by these results, we propose a simple model of coevolutionary prisoner's dilemma and study the effects of partner switching on stabilization of cooperation. Here, we concentrate on partnerships (links) instead of individuals. Because of reciprocity [43], CC relationships (mutual cooperation) are most likely to be bilaterally accepted. On the other hand, CD and DD relationships are prone to break up since (i) in the former case the C may not bear the exploitation of the neighboring D and (ii) in the latter case both D's would prefer to avoid unfavorable ties and search for a C to exploit. It should be noted that, in Refs. [31,32], the authors found cooperation can be systematically increased when only DD links can be rewired. Therefore one might expect that when both CD and DD links are permitted to switch, cooperation would achieve higher levels. However, the situation

———————
*fengfu@fas.harvard.edu
†wute@pku.edu.cn
‡longwang@pku.edu.cn

is complicated because *D*'s try to terminate interactions with themselves and search for a *C* to exploit, while *C*'s simultaneously struggle to avoid disadvantageous interactions with *D*'s. Therefore, this leads to an "arms race" between *C*'s and *D*'s in their search for new partners. For simplicity, we will first pay attention to the simplified model in which only *C* can unilaterally dismiss an unfavorable partnership and then randomly chooses a member from the rest of the population (excluding its immediate neighbors) as its future partner. Later, we will extend our model to allow all individuals to switch their defective partners (i.e., both *CD* and *DD* links can be rewired). We update individuals' strategies in a manner similar to the "pairwise comparison rule" [44] by randomly choosing a link. We should point out that this link-based updating diminishes the influence of large-degree nodes in their neighborhood [45].

The outcome of the coevolutionary dynamics hinges crucially on the ratio between the time scales of network adaptation (linking dynamics) and strategy dynamics. Reference [35] investigated the limit case where linking dynamics is much faster than strategy dynamics, leading to a rescaling of the original payoff matrix. Reference [34] numerically studied the threshold of how promptly an individual adjusts adverse ties for cooperation to thrive. Indeed, as the time scale ratio of link to strategy dynamics increases from 0 to ∞, the interplay between these two dynamical processes leads to a progressive crossover between pure linking dynamics and strategy dynamics on fixed graphs. However, to our knowledge, the full account of this transition has yet to be answered. In this paper, we aim to address this issue. Interestingly, our present work shows that cooperation levels may peak at a specific degree of the interplay between these two processes.

The rest of this paper is organized as follows. Section II briefly introduces the model. Section III describes the pair approximation method for studying the coevolutionary dynamics of the model. Section IV presents and discusses the simulation results. We end the paper with concluding remarks in Sec. V.

II. A MINIMAL MODEL

Let us briefly introduce the minimal model where only *CD* links can be severed. Initially, a total of *M* links pair the *N* individuals at random. Each individual has an equal probability of being a cooperator (*C*) or defector (*D*). As opposed to the node-based updating mechanism commonly adopted in previous studies [25], here we use a link-based update rule. In each time step, we randomly pick a link that connects a pair of players with different strategies, i.e., a *CD* link denoted by E_{ij}, to update. With a given probability *w*, the individuals *i* and *j* connected by the link E_{ij} update their strategies; otherwise, E_{ij} is rewired (with probability $1-w$). Note that the parameter *w* here exactly controls the extent of the interplay between strategy updating and network rewiring.

When a strategy updating event occurs, the individuals *i* and *j* play the prisoner's dilemma with all their social partners (those directly connected to them) and accumulate a total payoff P_i and P_j, respectively. Following previous com-

mon practice [12,46], the payoff matrix can be rescaled as

$$
\begin{array}{c}
\begin{array}{cc} C & D \end{array} \\
\begin{array}{c} C \\ D \end{array}
\begin{pmatrix} 1 & 0 \\ 1+u & u \end{pmatrix},
\end{array}
$$

where $u \in (0,1)$ represents the cost-to-benefit ratio. Then the strategy of individual *j* replaces *i*'s strategy with a probability given by the Fermi function [12,47,48]

$$
\phi_{s_j \to s_i} = \frac{1}{1 + \exp[\beta(P_i - P_j)]}, \tag{1}
$$

where $\beta \in [0, \infty)$ denotes the intensity of selection ($\beta \to 0$ leads to random drift, while $\beta \to \infty$ leads to the deterministic imitation dynamics). Otherwise, the strategy of *i* replaces the strategy of *j* with likelihood $\phi_{s_i \to s_j} = 1 - \phi_{s_j \to s_i} = \frac{1}{1+\exp[\beta(P_j - P_i)]}$.

When link E_{ij} is rewired (that is, partners are switched), the *C* player unilaterally dismisses the partnership with the *D* neighbor, and then randomly picks a player from the remainder population as its new partner. Only cooperators are allowed to unilaterally dismiss the adverse partnership and to seek a new one. In other words, cooperators impose a sort of conditional interaction with their partners, whereas defectors are at an advantage irrespective of the strategies of their partners. Even if a defector wants to further improve its payoff by dumping its defective opponent, it is barely able to establish new long-term partnerships under mutual agreement. For simplicity, we only permit cooperators to break adverse partnerships in this minimalist model. Later, we shall investigate the extended situation where both *CD* and *DD* links can be severed. It is worth noting that the total number of links remains unchanged during coevolution, resulting in a conservation in the average degree of the evolving partner network. In what follows, we mainly investigate the trade-off between strategy dynamics and partner network adaptation (i.e., the impacts of parameters *u* and *w* on the evolution of cooperation).

III. PAIR APPROXIMATION

We can first study the coevolutionary dynamics of the minimal model using pair approximation [49–52]. Originally, this method of extended dynamical mean-field approximation was suggested by Dickman [49,50]. The dynamic state and the topological structure of the network can be described in terms of the mean field quantities N_C, N_{CC}, and N_{DD}, where N_C denotes the number of cooperators and N_{CC} (N_{DD}) the number of *CC* (*DD*) links in the network. To describe the time evolution of these variables we need to use the moment closure approximation [51] when counting the number of all triplets N_{XYZ} in the network, connecting individuals with the respective states $X, Y, Z \in [C, D]$. We approximate N_{XYX} as the product of the number of *XY* links N_{XY} and the average number of *YX* links $\frac{N_{YX}}{N_Y}$ that a given type of *Y* has. That is, $N_{XYX} = N_{XY} \frac{N_{YX}}{N_Y}$ and $N_{XXY} = N_{XY} \frac{2N_{XX}}{N_X}$. Accordingly, we obtain a system of three coupled ordinary differential equations as follows:

$$\dot{N}_C = wN_{CD}\tanh\left[\frac{\beta}{2}(\bar{\pi}_C - \bar{\pi}_D)\right], \qquad (2)$$

$$\dot{N}_{CC} = w\left(N_{CD}\phi_{C\to D} - 2N_{CD}\frac{N_{CC}}{N_C}\phi_{D\to C} + N_{CD}\frac{N_{CD}}{N_D}\phi_{C\to D}\right)$$

$$+ (1-w)\frac{N_C}{N}N_{CD}, \qquad (3)$$

$$\dot{N}_{DD} = w\left(N_{CD}\phi_{D\to C} - 2N_{CD}\frac{N_{DD}}{N_D}\phi_{C\to D} + N_{CD}\frac{N_{CD}}{N_C}\phi_{D\to C}\right). \qquad (4)$$

Equation (2) describes the change of N_C due to strategy updating, where the average payoff of a C player is $\bar{\pi}_C = R\frac{2N_{CC}}{N_C} + S\frac{N_{CD}}{N_C}$ and that of a D-player is $\bar{\pi}_D = T\frac{N_{CD}}{N_D} + P\frac{2N_{DD}}{N_D}$. The first term in Eq. (3) corresponds to the change of N_{CC} as a result of strategy dynamics while the second term represents the conversion of CD links into CC links by rewiring. Equation (4) accounts for the change of N_{DD} due to strategy dynamics. The numbers of individuals and links are constant during the course of coevolution, i.e., $N_C + N_D = N$ and $N_{CC} + N_{CD} + N_{DD} = M$, so Eqs. (2)–(4) are closed and can be numerically solved.

IV. RESULTS AND DISCUSSION

In our simulations, we start from a population of individuals occupying the vertices of a random graph (i.e., the who-meets-whom relationship is random). There are no empty sites. Initially, 50% of cooperators are randomly distributed in the population. The population is of size $N = 1000$ and there are $M = 5000$ links (i.e., the average degree $z = 10$). The selection pressure is $\beta = 30$ (strong selection). Our results reported here are robust with respect to variations of the above model parameters. In the network adaptation process, we allow the network to be disconnected but prohibit duplicate links [53]. In the minimal model, we stop the simulations when no CD links are present in the network. In the extended model, the simulations are set to stop after 2×10^6 time steps. In our results, the fraction of cooperators is averaged over 1000 independent runs. We then investigate the effects of the interplay between strategy dynamics and partner switching on the evolution of cooperation and on the topological structures of the resulting partner network.

Let us start with the results of the minimal model. Figure 1 shows the cooperation level as a function of the cost-to-benefit ratio u for different w values. As $w \to 0$, individuals neither play games with their partners nor change their strategies; instead, cooperators dismiss their partnerships with defectors. Thus, the cooperation level remains at 0.5 irrespective of the specific u values. On the other hand, for $w \to 1$, the dynamics reverts to those of games on static graphs. However, for intermediate w, strategy dynamics and partner switching occur simultaneously: adaptive partner switching affects the partner network while the adjusted partner network influences the strategy dynamics. This kind of close feedback loop stabilizes cooperation. In this situation, the
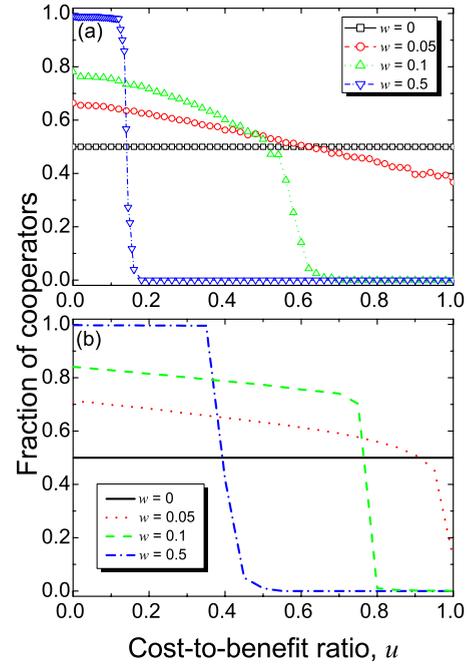
FIG. 1. (Color online) Evolution of cooperation. Fraction of cooperators as a function of cost-to-benefit ratio u with different w values: (a) simulations and (b) pair-approximation results.

cooperation level gradually decreases with increasing u, indicating that larger temptation to defect (i.e., larger u value) makes it harder for cooperation to evolve, even though cooperators are privileged to switch their defective partners. Nevertheless, for small w, that is, when individuals adjust their partners much faster than they adapt their strategies, persistent cooperation can be found in the entire region of $u \in [0,1]$ [Fig. 1(a)]. For increasing w, the transition from the dominance of cooperators to their extinction becomes sharper as u increases. At $w = 0.5$, for instance, the cooperation level steeply transits from 1 to 0 at $u \approx 0.12$. Additionally, the pair-approximation predictions qualitatively agree with our simulations [Fig. 1(b)]. These results suggest a nontrivial trade-off between strategy dynamics and the partner switching process.

To understand such interplay, we investigate the cooperation level as a function of the probability of strategy dynamics, w, corresponding to different u values (Fig. 2). Interestingly, the impact of parameter w acts differently on cooperation depending on the specific u value. At small u values (e.g., $u = 0.01$ in Fig. 2), the cooperation level monotonically increases with increasing w until a plateau of high cooperation is reached. With increasing u, the length of the plateau decreases. After this plateau, the cooperation level decreases as w approaches 1. As a consequence, for large u, the cooperation level peaks at specific values of w. As shown in Fig. 2, an optimal cooperation exists at $w \approx 0.3$ for $u = 0.2$. For larger u close to 1, the cooperation level monotonically decreases with increasing w. In this case, cooperation is only maintained when w is small—that is, cooperators are only able to survive when they promptly cut down the unfavorable interactions with defectors. Figures 1 and 2 demonstrate that the mutually interactive driving forces, strategy
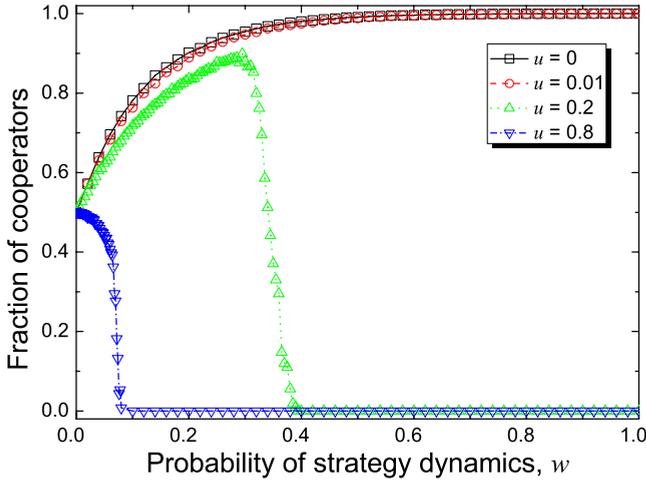
FIG. 2. (Color online) Fraction of cooperators as a function of w with different u values.

FIG. 4. (Color online) Number of isolated defectors as a function of w at final state.

adaptation and partner adjustment, play a crucial role in determining the evolutionary fate of cooperation.

Further, it is worthwhile to explore the topological structures of the resulting partner network. Figure 3 shows the respective degree distributions of cooperators and defectors corresponding to different u and w values. In our minimal model, because of the partner switching process, the partner network tends to divide into two major communities of C and D, along with a few isolated D's. Noteworthy, this fragmentation of the partner network to the largest extent enhances the assortment between C's which plays a crucial role in the maintenance of cooperation [6,7]. At relatively mild conditions for cooperation [e.g., for $u=0.2$ in Figs. 3(a) and 3(b)], the size of the cooperator community is much larger than that of the defectors in the case of small w (0.1 and 0.3). With $u=0.2$ and $w=0.1$, the degree distribution of cooperators and defectors both approximately follow a Poisson distribution. Most defectors have few partners. When $u=0.2$

and $w=0.3$, the number of defectors is significantly reduced; in other words, there is a significant increase in cooperators with a large number of partners. Notably, there are still a few surviving defectors who maintain a moderate number of partnerships with themselves. Under harsh conditions for cooperation (i.e., large u), in contrast, only when surviving cooperators successfully dump all their defective neighbors instead of becoming defectors themselves [e.g., for small value of $w=0.1$ shown in Fig. 3(c)], cooperation persists. In this case, the average number of partners a cooperator has is still much larger than that of defectors, but cooperation fails to become a dominant trait in the population. Not surprisingly, for larger w [$w=0.3$ in Fig. 3(d)], cooperators eventually vanish during coevolution and the emergent network shows a Poisson distribution with reduced heterogeneity. By comparing Figs. 3(a)–3(d), we can conclude that, depending on the strength of the dilemma (i.e., u) and the promptness of partner switching (i.e., $1-w$), the coevolutionary dynamics shapes the topologies of the resulting partner network.

Note that in Figs. 3(a)–3(c) there exist a few isolated defectors, while in Fig. 3(d) there are not any. To more explicitly describe this feature, we plot in Fig. 4 the number of isolated defectors as a function of w with different u values. We find that, for small u, the number of isolated defectors is a one-humped function of w. There is an intermediate w which leads to a maximum number of isolated defectors. Furthermore, the isolated defectors exist as long as w is below a certain threshold. However, for large u, the number of isolated defectors is a decreasing function of w and comes to 0 when w exceeds a certain threshold. Such thresholds correspond to the situations where the system converges to an absorbing state (full C or D; see Fig. 2).

Thus far, we have studied the evolution and structural organization of cooperation under a partner-switching mechanism. We would like to discuss some implications from our results obtained in the minimal model. In the present study, the interactions and competition between individuals are directed by the underlying partner network. Accordingly, in order for cooperation to spread widely, it is intuitive that a seemly balance between partner switching

FIG. 3. Degree distributions of the resulting partner networks. Black bars denote defectors and light gray ones cooperators.

and strategy adaptation is needed such that the cooperation level can be maximized before cooperators abstain from interacting with their defective partners. This scenario likely occurs at u values of no larger than 0.6. Otherwise, cooperation cannot have a chance to thrive, despite how swiftly cooperators adjust their unfavorable partners. Cooperators can punish their defective partners by leaving and seeking new potentially profitable ones. Such partner switching tends to enhance the assortment between cooperators as well as to increase (reduce) the average number of partners of cooperators (defectors), leading to a certain amount of degree heterogeneity. Such heterogeneity, if an individual's accumulated payoff is used in strategy dynamics, to some extent creates a "green house condition" for cooperation to flourish [34]. In addition, the community of cooperators can increase in size when outperforming neighboring defectors, as these periphery defectors are likely to turn into cooperators in strategy dynamics. However, neither of these two dynamics can independently enhance cooperation (i.e., at the two limit cases of $w=0$ and $w=1$). Thus, cooperation is induced by the interplay between strategy dynamics and partner network adaptation.

The emergent network degree heterogeneity and "network reciprocity" [54] are insufficient to offset the influences of the huge temptation to defect (i.e., large u) in strategy selection. In this situation, the size of the C community is inclined to shrink rapidly. Therefore, cooperation can be sustained only when cooperators abstain from interacting with defectors. In other words, cooperators' swift partner adjustment likely leads to the fragmentation of the partner network, where cooperators and defectors must reside in largely separated communities. In our present minimal model, we only consider the selection towards individuals through strategy dynamics. Furthermore, a higher level of selection, namely, group selection [55], ought to be in effect when the population breaks into C community and D community. In this case, the C community is more productive and more constructive than the D community, and so the D community is likely to be eventually replaced by the C community.

Recently, punishment was proposed to be an effective measure to enhance cooperation in some situations [56–58]. Generally, punishment is costly and winners don't punish [59]. Instead of adopting this type of "hard" punishment, here we implement a "soft" punishment by allowing individuals to terminate future interactions with their defective partners. As a consequence, a few isolated defectors may arise in the population. This is the severe case—ostracism [58]—in which those individuals lose all their partners and excluded from any group. Those banished defectors can be recruited by chance in the partner switching process, and learn to cooperate under the influence of the neighboring cooperators. In addition, some defectors with decreased number of partnerships readily shift to cooperation during strategy dynamics, avoiding further unfavorable outcomes of banishment. This represents a kind of social selection force, which can further stabilize cooperation. As a result, partner switching can enhance the level of cooperation that is achieved in our model.

Finally, let us take into account some possible extensions of our original model that better demonstrate the general role
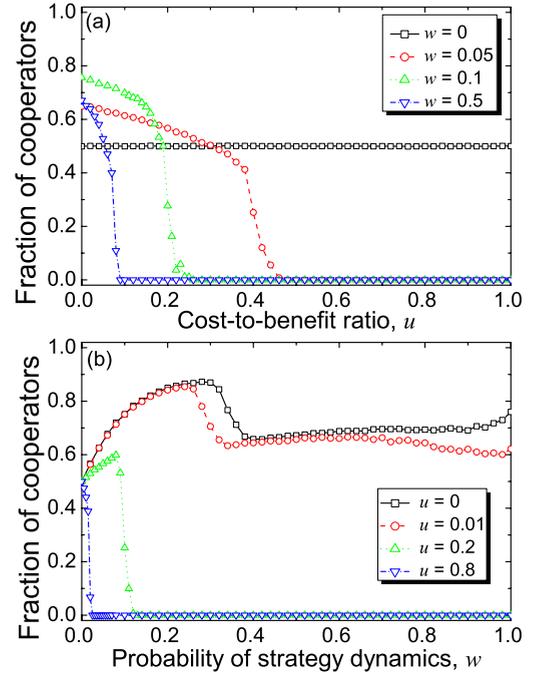
FIG. 5. (Color online) Evolution of cooperation when an individual's normalized payoff, other than accumulated one, is used in strategy dynamics. Fraction of cooperators as a function of (a) u and (b) w.

of partner switching in the stabilization of cooperation. It is reported that normalizing an individual's payoff, to a large extent, diminishes the positive role of network heterogeneity in promoting cooperation [22,23]. This observation has been made on fixed networks. Nevertheless, one may ask whether cooperation can still be promoted by a partner-switching mechanism if normalized payoff is adopted in the strategy updating rule. If so, what are the effects of using normalized payoff on cooperation in our case (i.e., on evolving networks)? To do this, we consider using the normalized payoff of an individual i, $\bar{P}_i = P_i/k_i$, where k_i denotes the number of partners of i. Accordingly, the strategy updating rule Eq. (1) becomes

$$\phi_{s_j \to s_i} = \frac{1}{1 + \exp[\beta(P_i/k_i - P_j/k_j)]}. \quad (5)$$

The corresponding results are summarized in Fig. 5. Due to normalization of an individual's payoff, the cooperation level is weakened as expected [see Figs. 1(a) and 5(a)]. Thus, this result is consistent with previous findings that cooperation is inhibited by using normalized payoff in heterogeneous networks [22,23]. On the other hand, the differences in the individuals' accumulated payoffs arising from network heterogeneity would help the evolution of cooperation. Moreover, we confirm that the stabilization impact of cooperation by partner switching remains efficacious [Fig. 5(b)], despite requiring cooperators to react more promptly to adverse ties (i.e., higher w). Noticeably, there still exists an optimal tendency to switch partners, resulting in the maximal cooperation level. Thus our results reported here are robust to the
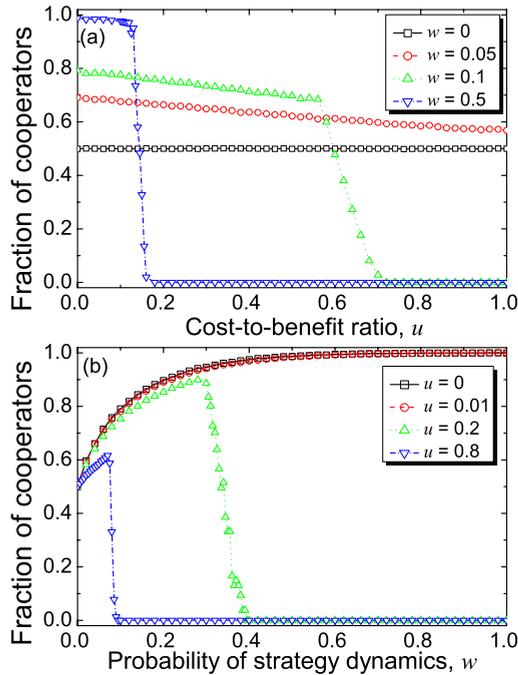
FIG. 6. (Color online) Evolution of cooperation under the situation where each individual can break up adverse partnerships (i.e., both CD and DD links can be rewired in partner-switching dynamics). Fraction of cooperators as a function of (a) u and (b) w.

variation in assessment of an individual's success (that is, success according to an accumulated payoff versus a normalized one).

In addition, let us extend the original model by allowing defectors to break up disadvantageous partnerships as well. Both CD and DD links can now be severed. In our simulations, a link is chosen at random to update: with probability w the strategy dynamics proceeds; otherwise the link, if it is CD or DD, is rewired with probability $1-w$. As mentioned before, the new link is only attached to the C if a CD link is severed; while for a DD link to be rewired, either of the two D's keeps the new link. We report the main results in Fig. 6. We can see that the cooperation level here is higher than in the original model, especially for small w [see Figs. 1(a) and 6(a)]. Remarkably, cooperation still can be enhanced at large cost-to-benefit ratio u, in sharp contrast to the original model [compare the curve for $u=0.8$ in Figs. 1(b) and 6(b)]. Prior to the present work, Refs. [31,32] showed that when unsatisfied D's dismiss the partnerships with each other, cooperation can be substantially promoted. Ironically, D's keep searching for new C partners to exploit, leading to their own demise. Moreover, as we have shown here, when both C's and D's are capable of switching defective partners, the survivability

of C's is further improved. In this case, D's are never able to establish social interactions under mutual agreement, while C's tend to form long-term profitable partnerships between each other. As a result, the social selection force greatly unfavors D in the partner market where everyone tends to choose C for future interactions. Due to the fierce competition between C and D in switching adverse ties (in terms of attachment and detachment), it is likely that the C's and D's coexist with sparse interactions between them, which is slightly different from the situation where C's may abstain from interactions with D's (namely, two separate communities arise) in the original model. Altogether, we confirm that the promotion of cooperation can be induced by allowing either only cooperators or all individuals to switch unfavorable partners.

V. CONCLUSION

In summary, we have presented a model of coevolutionary prisoner's dilemma in which individual strategy and partner network coevolve. We have shown that cooperation can be stabilized by means of prompt partner switching. Depending on the cost-to-benefit ratio u specifying the strength of the dilemma, there exists an optimum tendency of switching defective partners (i.e., the parameter w), that maximizes the number of cooperators in the population. Furthermore, partner switching may enhance interactions between cooperators, who on average engage in much denser interactions (larger average number of partners) with themselves than with defectors. We also provide an extended pair approximation which accounts for the coevolutionary dynamics on a mean-field level. Complementing previous theoretical and numerical explorations [31–35], the present work provides a comprehensive account of the interplay between strategy dynamics and partner adjustment, showing that cooperation can be maximized by a certain degree of tradeoff between these two dynamical processes. We hope that our results offer insight into the understanding of the evolution of cooperation on dynamical population structures.

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